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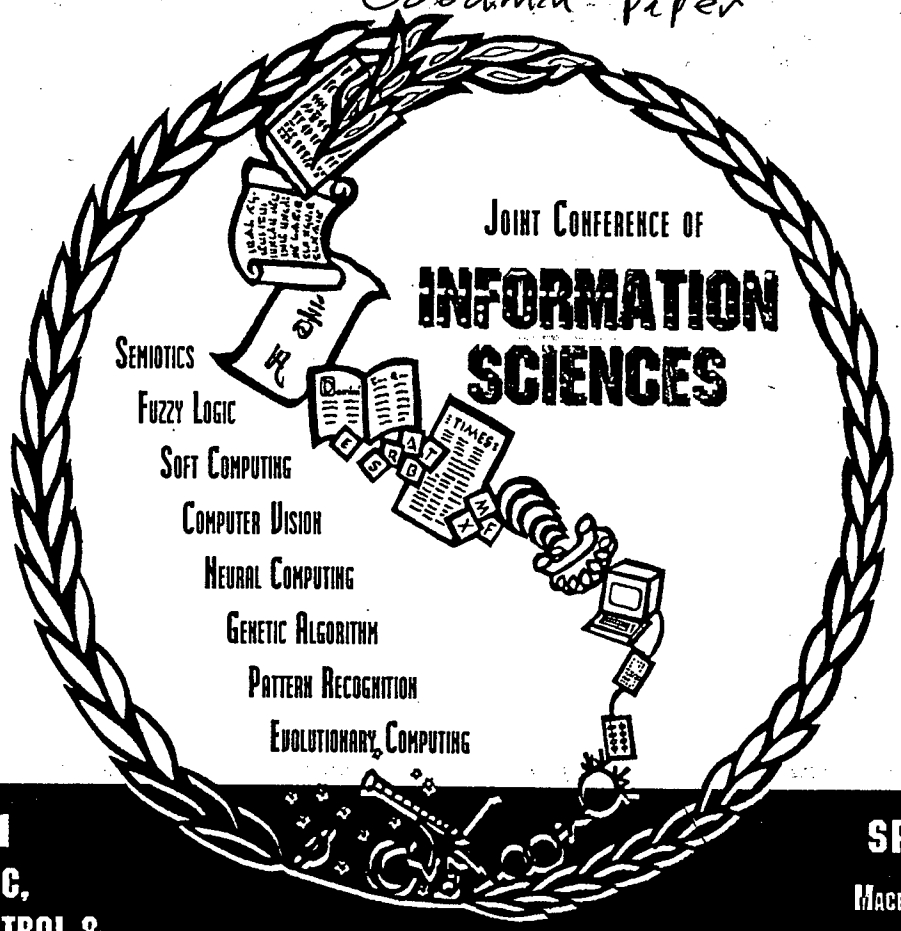

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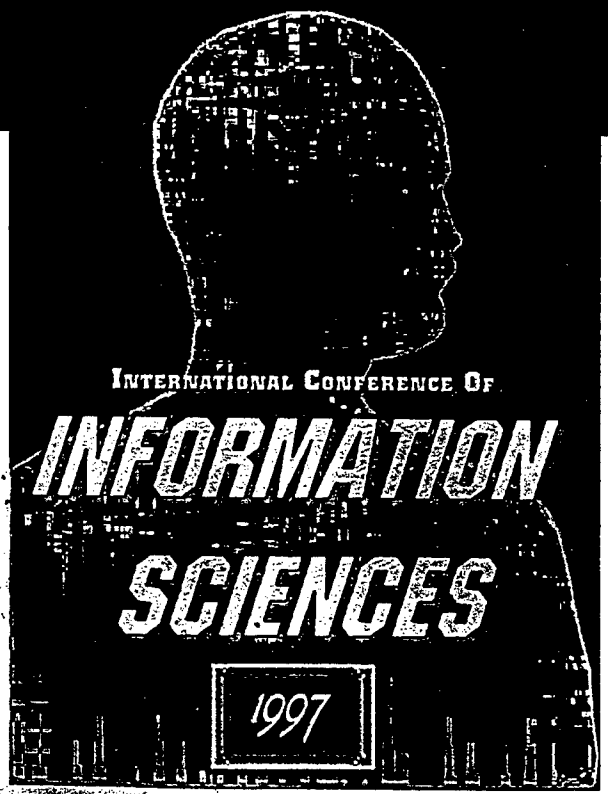

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HOMOMORPHIC-LIKE RANDOM SET REPRESENTATIONS FOR FUZZY LOGIC MODELS USING EXPONENTIATION WITH APPLICATIONS TO DATA FUSION

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Extended Abstract

Multiple source information forms one of the key components of data fusion. Such information may emanate from various mechanical sensor sources such as radar or doppler systems, or it may derive from human-based sources, such as via expert opinion expressed through natural language; or the information may consist of both types of sources. In addition, in general associated with each unit of information is a degree of uncertainty / certainty, which in the case of mechanical-based sources, is determined through the use of probability. Probability also plays a role in the case of linguistic-based information, where uncertainty is often measured initially through the use of fuzzy logic. This is because it is now well-established that a sound identification between a central aspect of fuzzy logic and probability descriptions is achievable through the use of *one-point coverage representations* of certain classes of random sets corresponding to the fuzzy sets [1]. Often, information uncertainty is provided through standard unconditional probabilities of certain events or cumulative probability distribution functions (or parameterized families of such). In this case, at least in theory, all relevant events are given explicitly in a boolean (or sigma-) algebra, so that the full probability space representing the uncertainty is known. Thus, one can evaluate probabilistically any desired logical combination of such events for use in decision-making, and in particular, in determining the degrees of similarity or consistency between these events by use of appropriately chosen metrics. The latter in a key way requires knowledge of the probability evaluations of logical conjunctions or disjunctions.

On the other hand, at times, information uncertainty is provided in a way that there appears to be no single underlying boolean event whose probability evaluation matches the prescribed uncertainty. Such uncertainty is often expressed in the form of particular given functions of probability evaluations of contributing simpler events. For example, when these functions are simple arithmetic division with arguments being pairs of events, each numerator argument event being a subevent of the denominator argument event, the uncertainty corresponding to each unit of information then becomes a conditional probability. However, in general, the standard development of probability theory and statistics has *not* produced a way to represent conditional probabilities as single event probability evaluations, so that the basic decision-making applications mentioned above for the unconditional case can not be applied analogously. Hence, e.g., one cannot use standard probability techniques in a systematic sound way to analyze quantitatively a collection of inference rules such as "if b, then a", "if d, then c",... for similarity or difference when the uncertainties associated with these rules are the conditional probabilities P(a|b), P(c|d),... . Some exceptions to this include the situation where all of the conditional probabilities represent independent information, or where all of them have an identical denominator, or where the conditional expressions

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are in a chained form. Recently, probability theory has been expanded to address the above problem through the use *of conditional event algebra* [2],[3].

Another situation where uncertainty does not seem to correspond to the probability evaluation of a single event occurs when the uncertainty is provided as a weighted sum of probabilities of more elemental events which may be overlapping (so that the total probability expansion theorem which critically depends on disjointness is not applicable). This can arise when experts are combining probabilities subjectively or, via the connection of probability with fuzzy logic mentioned above, through the use of fuzzy partitions. Similarly, uncertainty may be provided as more complicated nonlinear functions of contributing event probabilities.

Apropos to the above discussion, the primary goal of this paper is to show how a relatively new mathematical tool, *relational event algebra* [4], an extension of conditional event algebra, can be utilized, in conjunction with one-point random set coverages, to represent a class of fuzzy logic models which uses exponentiation to model modifiers.

Consider first, for any positive integer m, and any choice of (boolean) event, say C, relative to a given probability space (Ω,B,P), the exponentiated probability $(P(C))^m$. This is naturally matched by the corresponding m-factor cartesian product evaluation:

$$(P(C))^m = P_0(C^m), \qquad (1)$$

where, by definition, the *m-exponentiation of event* C is

$$C^m = C \times ... \times C \quad \text{(m factors)} \qquad (2)$$

and P_0 is the m-product probability measure whose m marginal measures are identical to P. Next, consider the following sequence of results:

Theorem 1. (Goodman & Nguyen [2]) For any choice of probability space (Ω,B,P) corresponding to a sample space of unconditional events of interest, there is always an extension to a larger countable infinite product probability space (Ω_o,B_o,P_o) which also contains all *conditional events* (a|b) formed from all events $a,b \in B$, in the sense that independent of the choice of (well-defined) P,

$$P_o((a|b)) = P(ab)/P(b) = P(a|b) \text{ (cond. prob.)} \qquad (3)$$

Probability evaluations by P_o of any finite logical combination of such conditional events (a|b), (c|d),... for all a,b,c,d,... $\in$ B, are finitely computable and, under relatively weak independence assumptions, the above space can be shown to be unique [3]. ■

Theorem 2. (Goodman & Kramer [4]) Given any probability space (Ω,B,P), there exists a product probability space (Ω_1,B_1,P_1) associated with the conditional event extension (Ω_o,B_o,P_o) such that for any nontrivial event $C \in B$ with respect to P, i.e., $0 < P(C) < 1$, and for each choice of rational number $t \in [0,1]$, there is an (conditional) event $\theta(t) \in B_1$, independent of the particular choice of P -- and that of C in its behavior -- such that

$$P_1(\theta(t)) = t. \qquad (4)$$

Call any such event a *constant-probability* event. ■

Theorem 3. (Goodman & Kramer [4]) Consider any function $h:[0,1] \rightarrow [0,1]$ (where the endpoints may or may not be included) with

$$h(s) = (1-s)^m \cdot k(s), \qquad (5)$$

where $k:[0,1] \rightarrow [0,1]$ is an analytic function such that h takes the form of a finite polynomial or infinite series, given, without loss of generality, as

$$h(s) = \sum_{j=0}^{+\infty} (s^j \cdot (1-s)^m \cdot t_j), \text{ for all s in } [0,1]; \qquad (6)$$

t_j are all $\in [0,1]$ and m is a positive integer. Then, there is a natural corresponding event-valued function, denoted also by h, when no ambiguity is present (we have already used such notation above to denote exponentiation of events), such that for any choice of event C,

$$h(C) = \bigvee_{j=0}^{+\infty} ((C')^{m-1} \times C^j \times C' \times \theta(t_j)) \qquad (7)$$

is a disjoint series of product events, and for any choice of (nontrivial) P, slightly abusing notation,

$$P_1(h(C)) = \sum_{j=0}^{+\infty} (P_o(C^j) \cdot P_o((C')^m)) \, P_1(\theta(t_j)))$$

$$= \sum_{j=0}^{+\infty} ((P(C))^j \cdot (P(C'))^m \cdot t_j = h(P(C)). \qquad (8)$$

■

Also, noting the special role 0 and 1 play in probability, we may at times consider in place of the form in eq.(7) the more appropriate dual expansion of h in terms of $s' = 1-s$

$$h(s') = \sum_{j=0}^{+\infty} ((s')^j \cdot s^m \cdot t_j), \text{ for all } s \in [0,1], \qquad (9)$$

with corresponding event-valued form

$$h(C') = \bigvee_{j=0}^{+\infty} (C^{m-1} \times (C')^j \times C \times \theta(t_j)) \quad (10)$$

with compatibility eq.(8) now becoming for all P,

$$P_1(h(C')) = h(P(C')) = h(1-P(C)). \quad (11)$$

Thus, eqs.(1),(2) can be considered limiting cases of eqs.(10),(11).

Note the basic identity

$$s^{1.5} = (1-s')^{1.5} = s^2.(1-s')^{-0.5}, \text{ for all } s \in [0,1], \quad (12)$$

where now the right-hand factor function $(1-s')^{-0.5}$ as a function of s' indeed satisfies all of the requirements with infinite series form

$$(1-s')^{-0.5} = \sum_{j=0}^{+\infty} (s')^j.t_{j,0.5}, \text{ for all s in } [0,1], \quad (13)$$

with binomial coefficients in [0,1] being

$$t_{j,0.5} = \Gamma(0.5+j)/(\Gamma(0.5).j!) =$$

$$\begin{cases} 1, & \text{if } j = 0 \\ ((0.5)\cdot(1.5)\cdot\ldots\cdot(0.5+(j-1)))/j!, & \text{if } j = 1,2,3,\ldots \end{cases} \quad (14)$$

Then, we can finally define, for any event $C \in B$, the disjoint event series

$$C^{1.5} = \bigvee_{j=0}^{+\infty} (C \times (C')^j \times C \times \theta(t_{j,0.5})) \quad (15)$$

A straightforward probability evaluation of eq.(15) yields for any P, and corresponding P_1,

$$P_1(C^{1.5}) = (P(C))^{1.5}, \quad (16)$$

a special case of Theorem 3, compatible with the integer exponentiation of events considered earlier. Since the coefficients $t_{j,0.5}$ in the above series are all dominated by unity, this series converges at least as fast as the ordinary power series and thus reasonably small finite truncation approximations for all event values P(C) bounded away from zero are possible. These convergence rates can also be refined by noting the series given in eq.(13) is a particular member of a well-studied class of series [5]. (In fact, there appears to be a formal connection between our extending event exponentiation from the integral case to the full real number case and the concept of fractional derivatives.)

In any case, note the complement of $C^{1.5}$, up to P_1-probability one, is easily seen to be

$$(C^{1.5})' = \bigvee_{j=1} (C \times (C')^j \times C \times (\theta(t_{j,0.5}))') \vee C'. \quad (17)$$

In a similar vein, note the exponential integer power complements

$$(C^2)' = (C \times C') \vee C', (C^3)' = (C^2 \times C') \vee (C \times C') \vee C'. \quad (18)$$

Also, via eqs.(1) and (2), we have the natural orderings when $C > \emptyset$

$$\emptyset < C^3 < C^2 < C^{1.5} < C < \Omega. \quad (19)$$

Consider now the following simple-appearing example where two expert observers independently - or perhaps with some coordination or influence – provide their opinions concerning the same situation of interest: namely the description of an enemy ship relative to length and visible weaponry of a certain type. Suppose that experts 1 and 2 have both viewed the ship through fairly dense fog conditions twice. The first time, both only observe ship length and the second time both only observe potentility for weapons on-board. They then present their conservative (i.e., disjunctive logic) opinions:

α = "Ship A appears very long or at least it seems to have the capacity for quite a (i.e., between moderately and very) large number of q-type weapons being possibly on deck",

β = "Ship A may be extremely long or has the capacity for a moderate number of q-type weapons on deck" (20)

Identifying any fuzzy set membership function with its associated attribute and denoting its domain of values by D with appropriate subscripts, we consider first the relatively neutral attributes

d = moderate quantity of q-type weapons on deck,
g = moderately long. (21)

Let the variable x denote length of ship A and the variable y denote number of q-type weapons on-board A. Then, by use of exponentiation and some choice of fuzzy logic disjunction operators $\overline{\vee}_1$, $\overline{\vee}_1$, a model is proposed for each of the above two natural language expressions, where exponent 2 corresponds to "very", 3 to "extremely, 1.5 to "quite a" and 1 to "moderate", yielding finally

$$\alpha(x,y) = (g(x))^2 \,\overline{\vee}_1\, (d(y))^{1.5},$$
$$\beta(x,y) = (g(x))^3 \,\overline{\vee}_2\, d(y), \quad (22)$$

all $x \in D_g$, all $y \in D_d$.

Theorem 4. (New result) For the two fuzzy logic models above, assuming D_g and D_d are finite sets:

(i) Consider the joint one-point coverage type random sets $S_1(g), S_1(d), S_2(g), S_2(d)$, where

$$P(x \in S_j(g)) = g(x),\ P(y \in S_j(d)) = d(y), \quad (23)$$

all $x \in D_g$, all $y \in D_d$, $j=1,2$.

These joint random sets are equivalent to the corresponding ordinary set membership transforms,

$$Q = \{(\phi(S_1(g))(x), \phi(S_1(d))(y), \phi(S_2(g))(x), \phi(S_2(d))(y)): \text{all } x \in D_g, y \in D_d\} \quad (24)$$

as joint zero-one random variables, with hitting functions being g and d [1]. The value j=1 refers to the model for α and j=2 for β.

Then, for any choice of copula, cop, determining the joint distribution of Q, and hence, the joint distribution of the four one-point coverage type random sets, there is a corresponding copula cop_0, with corresponding cocopula denoted by $\overline{v}$, determining the joint distribution of the exponentiated random sets $(S_1(g))^2$, $(S_1(d))^{1.5}$, $(S_2(g))^3$, $S_2(d)$ (or, likewise, their equivalent joint zero-one random variables). cop_o has marginal copulas $cop_{o,j}$ with corresponding cocopula $\overline{v}_j$, $cop_{o,1}$ (or $\overline{v}_1$) determining the joint distribution of $(S_1(g))^2$, $(S_1(d))^{1.5}$ and $cop_{o,2}$ (or $\overline{v}_2$) determining the joint distribution of $(S_2(g))^3$, $S_2(d)$. Also, the following *homomorphic-like* relations hold, when the $\overline{v}_j$ used in eq.(22) are chosen as above:

$$\begin{aligned}\alpha(x,y) &= (P(x \in S_1(g)))^2\ \overline{v}_1\ (P(y \in S_1(d)))^{1.5} \\ &= P_1((x \in S_1(g))^2)\ \overline{v}_1 P((y \in S_1(d))^{1.5}) \\ &= P_1((x \in S_1(g))^2 \vee (y \in S_1(d))^{1.5}),\end{aligned}$$

$$\begin{aligned}t(\beta)(x,y) &= (P(x \in S_2(g)))^3\ \overline{v}_2\ P(y \in S_2(d)) \\ &= P_1((x \in S_2(g))^3)\ \overline{v}_2\ P(y \in S_2(d)) \\ &= P_1((x \in S_2(g))^3) \vee (y \in S_2(d)))\end{aligned} \quad (25)$$

for all $x \in D_g$, $y \in D_d$.

(ii) The specific relations between cocop and $\overline{v}_1$, $\overline{v}_2$ are given, for all $0 \le r,s \le 1$, as :

$$r\ \overline{v}_1\ s = 1 - f_1(r^{1/2}, s^{2/3}, cocop(r^{1/2}, s^{2/3})), \quad (26)$$

where

$$\begin{aligned}&f_1(r,s,cocop(r,s)) \\ &= ([(r+s-cocop(r,s))(1-cocop(r,s))] + \\ &\quad [(cocop(r,s) - r)(1-s)])[(1-s^{0.5})/(1-s)] \\ &+ ((cocop(r,s) - s)(1-r)) + 1-cocop(r,s), \quad (27)\end{aligned}$$

and

$$r\ \overline{v}_2\ s = 1 - f_2(r^{1/3}, s, cocop(r^{1/3}, s)), \quad (28)$$

where

$$\begin{aligned}&f_2(r,s,cocop(r,s)) \\ &= ((cocop(r,s) - s)r(1-r)) + ((cocop(r,s) - s)(1-r)) \\ &\quad + 1-cocop(r,s), \quad (29)\end{aligned}$$

with possible additional simplifications. ■

We write (with respect to an appropriate choice of a countable infinite factor product probability space), events

$$\begin{aligned}A(d,g;x,y;S_1) &= (x \in S_1(g))^2 \vee (y \in S_1(d))^{1.5} \\ &= (\underline{x} \in (S_1(g))^2) \times \Omega_1) \vee (\underline{y} \in (S_1(d))^{1.5}), \\ B(d,g;x,y;S_2) &= (x \in S_2(g))^3 \vee (y \in S_2(d)) \\ &= (\underline{x} \in (S_2(g))^3 \times \Omega_1) \vee (\underline{y} \in S_2(d)) \times \Omega_1).\end{aligned} \quad (30)$$

$$\underline{x} = (x,x,x,...),\ \underline{y} = (y,y,y,...), \text{ all } x \in D_g,\ y \in D_d. \quad (31)$$

Thus, Theorem 4 shows that the class of (boolean) events $A(d,g;x,y;S_1)$, $B(d,g;x,y;S_2)$, $x \in D_g$, $y \in D_d$, for S_1, S_2 arbitrary -- and hence for cop arbitrary -- naturally underlies the fuzzy logic model given in eq.(22). In turn, one can then measure the degree of similarity -- and test hypotheses for such -- utilizing, e.g., the probability distance $P(A\&B^*) + P(A'\&B)$ [4].

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